Geometria do espaço de gwistor

Rui Albuquerque[1]
[1]Universidade Évora

Email: rpa@uevora.pt

Resumo

O chamado "espaço de gwistor" foi descoberto em Portugal em 2006 inteiramente por matemáticos portugueses. Revelou-se, então, a existência de uma estrutura G_2 no fibrado de esferas tangente $\pi : SM \to M$ de qualquer dada variedade riemanniana M, orientável e de dimensão 4.

Recordemos que G_2 é caso excepcional entre as poucas classes de grupos de Lie simples que existem, dando origem a uma geometria excepcional muito em voga em Geometria e Física. É também um dos poucos grupos de Lie que aparecem como grupos de holonomia irredutível de variedades riemannianas não simétricas (a lista é dada por famoso resultado de M. Berger de 1955). Mas $G_2 \subset SO(7)$ só pode aparecer como grupo de holonomia, e R. Bryant provou mesmo que aparece, em variedades de dimensão 7 (claramente a dimensão de SM).

A representação de G_2 mais acessível é a de grupo dos automorfismos dos octoniões $\mathbb{O}$. Temos $SO(7)$ actuando na parte imaginária $\mathbb{R}^7$ de $\mathbb{O} = \mathbb{H} \oplus e\mathbb{H}$. A soma directa anterior, que generaliza por exemplo $\mathbb{C} = \mathbb{R} \oplus i\mathbb{R}$, revela já a construção de uma estrutura octoniónica em cada espaço tangente

$$T_u(TM) \simeq T_xM \oplus T_xM, \quad \forall u \in SM,\ x = \pi(u).$$

A técnica matemática envolvida é a dos espaços de twistor, e daí, usando outra bem comum para as letras, surgiu o nome gwistor.

Contamos apresentar os resultados iniciais, com ligação às variedades de Einstein, assim como os das últimas publicações.

Counting maps from curves to projective spaces via graph theory

Ethan Cotterill[1]

[1]CMUC, Universidade Coimbra

Email: cotterill.ethan@gmail.com

Resumo

Understanding when an abstract complex curve C of genus g comes equipped with a non-degenerate degree-d morphism to $\mathbb{P}^r$ is a fundamental question for curve theory. When C is general in the moduli space $\mathscr{M}_g$, and answer is provided by the celebrated *Brill–Noether theorem*, which establishes that the space of morphisms on C behaves as one would expect on the basis of linear algebra.

On the other hand, the "dual graph" construction provides a bridge between the moduli space of curves and the moduli space of (stable) metric graphs. This leads to the question of which stable graphs are general in a Brill–Noether sense. In work in progress with Melo, Neves, and Viviani, we study a seemingly-remarkable family of examples of graphs that decompose as triples of trees rooted a set of d common vertices. Here we will describe some of the combinatorics involved, and how these graphs relate to the configuration spaces $\mathscr{M}_{0,n}$ of n points on $\mathbb{P}^1$.

Toric spectral results

Rosa Sena-Dias [1]
[1]IST, Univ. Téc. Lisboa

Email: rsenadias@math.ist.utl.pt

Resumo

How many two dimensional convex polytopes are there with given side length and side direction? We will show that this simple question encapsulates some interesting geometry and is related to the following question: "Does the spectrum of the Laplacian on a toric manifold determine the toric manifold itself?". In this talk we hope to illuminate some of the more obscure words in the above question, explain why the answer is sometimes yes (with caveat) and why that is interesting. This is joint work work with Emily Dryden and Victor Guillemin.

Monads on multiprojective spaces

Helena Soares[1]
[1]ISCTE, Lisboa

Email: helena.soares@iscte.pt

Resumo

Given a smooth projective variety X, a *monad* on X is a complex

$$M_\bullet: \quad 0 \longrightarrow A \xrightarrow[\alpha]{} B \xrightarrow[\beta]{} C \longrightarrow 0$$

of coherent sheaves on X, with α an injective map and β surjective. The coherent sheaf $E := \ker\beta/\mathrm{im}\alpha$ is called the *cohomology sheaf* of the monad $M_\bullet$.

Monads were introduced by Horrocks in the sixties and since then they have proved very useful objects for constructing vector bundles and studying their properties.

When studying monads, one natural problem is to determine their existence. We construct a family of monads on the multiprojective space $X = \mathbb{P}^l \times \mathbb{P}^m$ of type

$$0 \to \mathscr{O}_{\mathbb{P}^l\times\mathbb{P}^m}(-1,-1)^a \to \mathscr{O}^b_{\mathbb{P}^l\times\mathbb{P}^m} \to \mathscr{O}_{\mathbb{P}^l\times\mathbb{P}^m}(1,1)^c \to 0$$

and give a cohomological characterisation of torsion-free sheaves that are the cohomology of monads of this form.

The abelianized Kummer surface

Thomas Baier [1]
[1]Universidade Porto

Email: tbaier@fc.up.pt

Resumo

For curves of genus 2, Hitchin's connection on the moduli space of vector bundles of rank 2 with trivial determinant requires a special construction (carried out by van Geemen and de Jong). In particular the geometry of the Kummer surface associated to the base curve plays a fundamental role in it. I plan to discuss the link between this classical topic and contemporary research concerning the monodromy of Hitchin's connection.

New tools for classifying Hamiltonian circle actions with isolated fixed points

Silvia Sabatini[1]
[1]EPFL, Lausanne

Email: silvia.sabatini@epfl.ch

Resumo

In 2009 Tolman formulated the "symplectic generalization of Petrie's conjecture": given a compact symplectic manifold M with a Hamiltonian S^1 action and minimal number of fixed points, is it possible to characterize all the possible cohomology rings and Chern classes that can arise? We turn this question into a computational problem, in the following way.

We derive a simple algebraic identity involving the first Chern class. This enables us to construct an algorithm to obtain *linear relations among the isotropy weights* at the fixed points. Since determining the weights at the fixed points determines the (equivariant) cohomology ring and Chern classes, this allows us to give a (positive) answer to the question.

In particular, we give a complete list of cohomology rings and Chern classes when $\dim(M)$ is less than or equal to 6, recovering the results obtained by Tolman, and in dimension 8 we prove that if the action extends to a Hamiltonian T^2 action, or if none of the weights is one, then the equivariant cohomology ring and Chern classes agree with the ones of $\mathbb{C}P^4$. This is joint work with L. Godinho, IST.

On Jaeger's HOMFLY-PT expansions, branching rules and link homology: a progress report

Pedro Vaz [1]
[1]IST, Univ. Téc. Lisboa

Email: pedro.forte.vaz@ist.utl.pt

Resumo

In 1989 François Jaeger showed that the Kauffman polynomial of a link L can be obtained as a weighted sum of HOMFLYPT polynomials on certain links associated to L. In this talk I will explain how to obtain more general expansions using the so called "branching rules" for Lie algebra representations and describe a program to obtain categorified versions of Jaeger's expansions for link homology theories (partially joint with E. Wagner).

Waldhausen decomposition of the complement of a link and systems of PDE's

Pedro Cristiano Silva[1]
[1]CEF/ISA, Univ. Téc. Lisboa

Email: pcsilva@isa.utl.pt

Resumo

We present a notion of rigid local system on the complement of a plane curve that generalizes the classical notion of rigid of local system in the Riemann sphere, due to Nicholas Katz and Pierre Deligne. The Waldhausen decomposition of the complement of an algebraic knot is a main ingredient of our definition. As a consequence we solve a problem considered by Sato, Kashiwara et al, by constructing a non trivial class of linear systems of PDE's "without accessory parameters". This is a joint work with Orlando Neto.

Abelian vortices and maps to projective space

João Baptista[1]
[1]IST, Univ. Téc. Lisboa

Email: joao.o.baptista@gmail.com

Resumo

Let M be a compact Kähler manifold. The space $\mathscr{H}$ of holomorphic maps from M to projective space has a natural compactification in gauge theory: a vortex moduli space $\mathscr{M}_v$. Both these spaces have natural, though distinct, L^2-metrics. I will show that when M is Riemann surface or a Grassmannian one can explicitly describe $\mathscr{M}_v$ and compute its total volume. This will then lead to conjectural formulae for the volume of $\mathscr{H}$.